A Solution Sketch For Sample Midterm

1. (a) Sequence of nodes that are expanded: A C F B E J. Solution path: A B E J. Path cost = 34. (b) Sequence of nodes that are expanded: A C F B G N. Solution path: A C G N. Path cost = 31. (c) Let's denote by $h^*(n)$ the path cost of the optimal solution from node n to a goal node. For h to be admissible, you need to show that $h(n) \leq h^*(n)$ for every node in the tree. This can be done by just enumerate them all: $h(A) = 25$, $h^*(A) = 31$, $h(B) = 26$, $h^*(B) = 31$, ...

2. The MAX play will choose the left move if:

$$min(max(x, 0), 10) > min(2, max(3, y)),$$

that is, if $x > 2$.

3. (a) "there is someone who respects everyone". (b) "everyone is respected by someone". (c) they are not equivalent. To explain why formally, you need to construct an interpretation that satisfies one but not the other. Since we did not talk much about formal semantics, you can just give any informal reasons.

4. (a) There are several ways to do this. Recall that when we transform a sentence to its conjunctive normal form, we said that all steps preserve logical equivalence except the step of Skolemization. So one possible way to show that these two sentences are equivalent is to see whether we can transform them into their CNF without using Skolemization. This is not always possible, but for this problem it is ok:

The first sentence:

$$\begin{array}{l}
\forall x\ [\exists y\ Loves(y,x)] \Rightarrow Happy(x) \mapsto \text{ (eliminate implication)} \\
\forall x\ \neg[\exists y\ Loves(y,x)] \lor Happy(x) \mapsto \text{ (move negation inside)} \\
\forall x\ [\forall y\ \neg Loves(y,x)] \lor Happy(x) \mapsto \text{ (move quantifiers to the front)} \\
\forall x,y\ \neg Loves(y,x) \lor Happy(x) \mapsto \text{ (drop quantifiers)} \\
\neg Loves(y,x) \lor Happy(x) \quad (1)
\end{array}$$

The second sentence:

$$\begin{array}{l}
\forall x,y\ Loves(y,x) \Rightarrow Happy(x) \mapsto \\
\forall x,y\ \neg Loves(y,x) \lor Happy(x) \mapsto \\
\neg Loves(y,x) \lor Happy(x) \quad (2)
\end{array}$$

Since (1) and (2) are the same (in general, it's ok if they can be made the same by renaming some variables), so they are equivalent.

A more general way of proving equivalence is to use resolution: to show that these two sentences are equivalent, you need to show that

$$[\forall x\ [\exists y\ Loves(y,x)] \Rightarrow Happy(x)] \Leftrightarrow [\forall x, y\ Loves(y,x) \Rightarrow Happy(x)]$$

is valid, that is, follows from the empty KB. By refutation using resolution, first transform the negation of the above equivalence into CNF, and then use resolution to derive the empty clause.

6. (a) A search strategy is complete if it is guaranteed to return a solution if one exists. (See page 73 in the textbook) (b) An inference procedure is complete if it can derive all conclusions entailed by the KB. (See page 286 of the textbook.)